The rank histogram

A versatile tool for forecast verification

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ECMWF

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Outline

1. The ensemble rank histogram
2. A short literature review
3. The observation rank histogram
4. The 2D ensemble rank histogram

1. The ensemble rank histogram

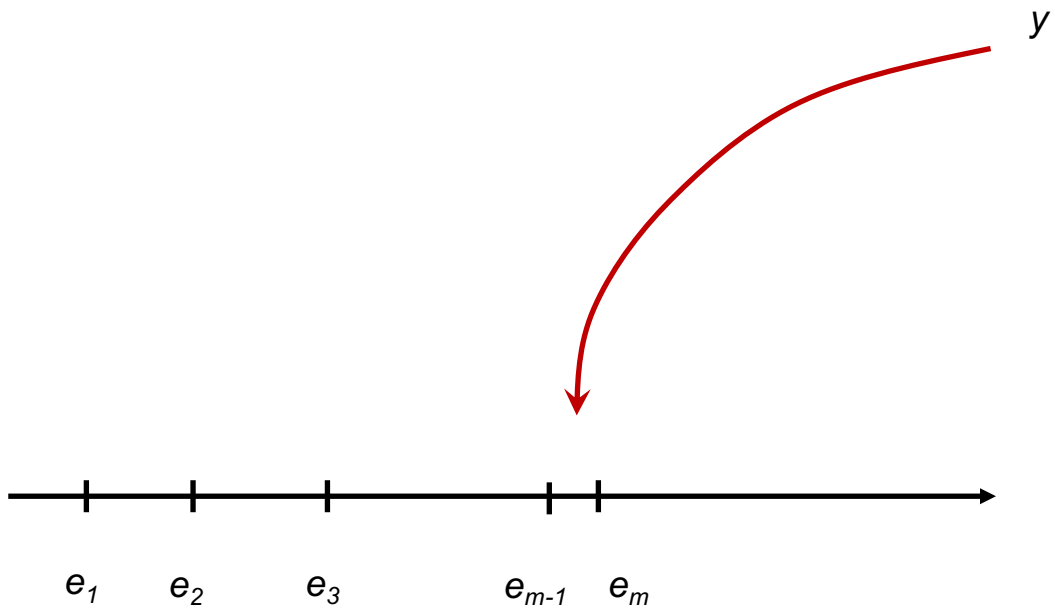
The rank histogram

a tool for comparing samples from 2 distributions.

The ensemble rank histogram

$e_1, e_2, \ldots, e_m$: sorted forecasts over m ensemble members at point (t,i,j)
y: observation corresponding to forecast e

for all t,s



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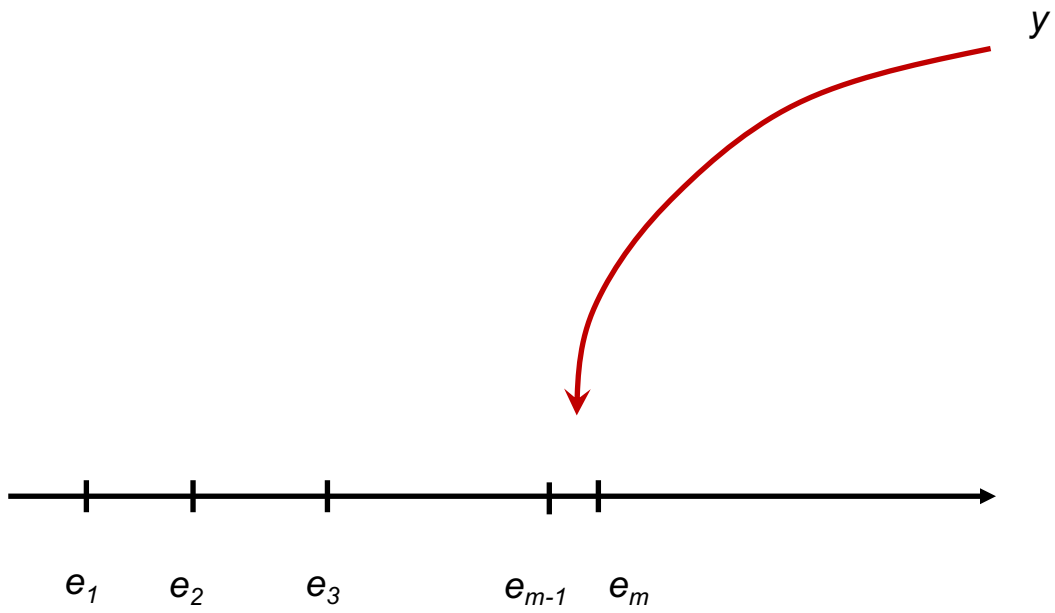

Complementary tool:
spread/skill diagram

2. A short* literature review

* and subjective

Hamill, T. M., **2001**: Interpretation of Rank Histograms for Verifying Ensemble Forecasts. *Mon. Wea. Rev.*, 129, 550–560

Keller, J.D., Hense, A., **2011** : A new non-Gaussian evaluation method for ensemble forecasts based on analysis rank histograms. *Met. Zet*. 20, 107-117

Bröcker J, Ben Bouallègue Z., **2020**: Stratified rank histograms for ensemble forecast verification under serial dependence. *QJR Meteorol Soc*. 146: 1976-1990.

Allen, S., Ziegel, J., Ginsbourger, D., **2024**: Assessing the calibration of multivariate probabilistic forecasts. *QJR Meteorol Soc*., 1–21.

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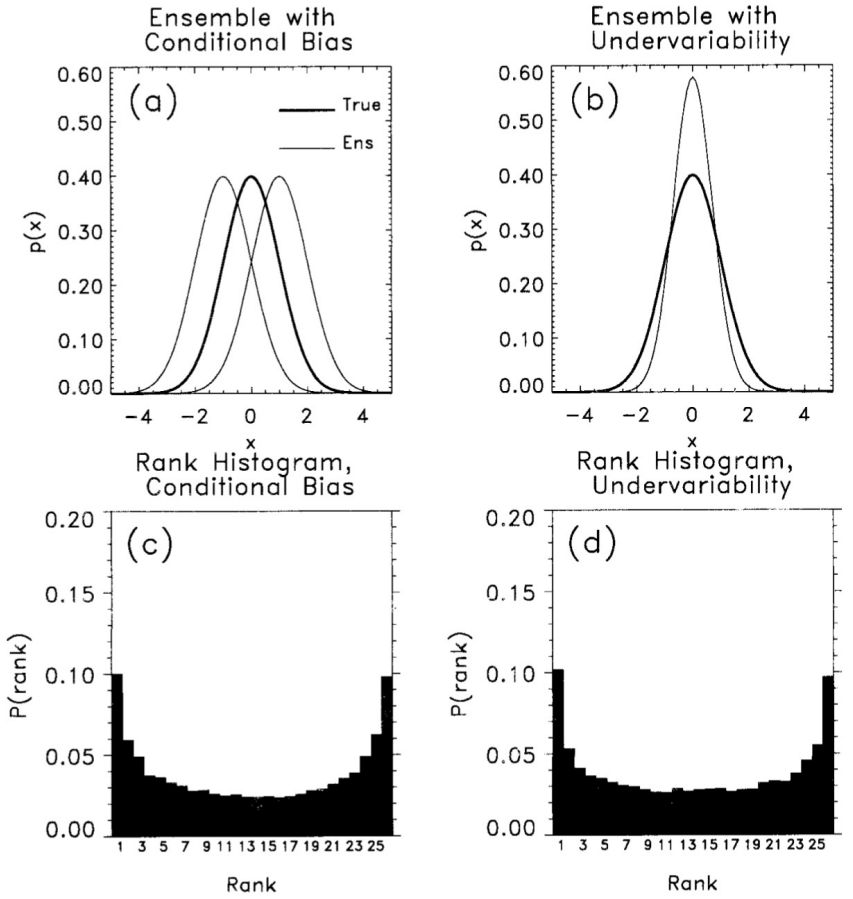

FIG. 4. (a) As in Fig. 2a, but where ensemble is selected with equally likely probability from one of the two biased distributions, a $N(-1, 1)$ or $N(1, 1)$ distribution, with the verification tallied 10 000 times for each distribution. (b) As in (a), but where ensemble forecasts are selected from a probability distribution with a lack of variability, $N(0, 0.69)$. (c) Rank histogram corresponding to (a). (d) Rank histogram corresponding to (b). Verification rank tallied 20 000 times.

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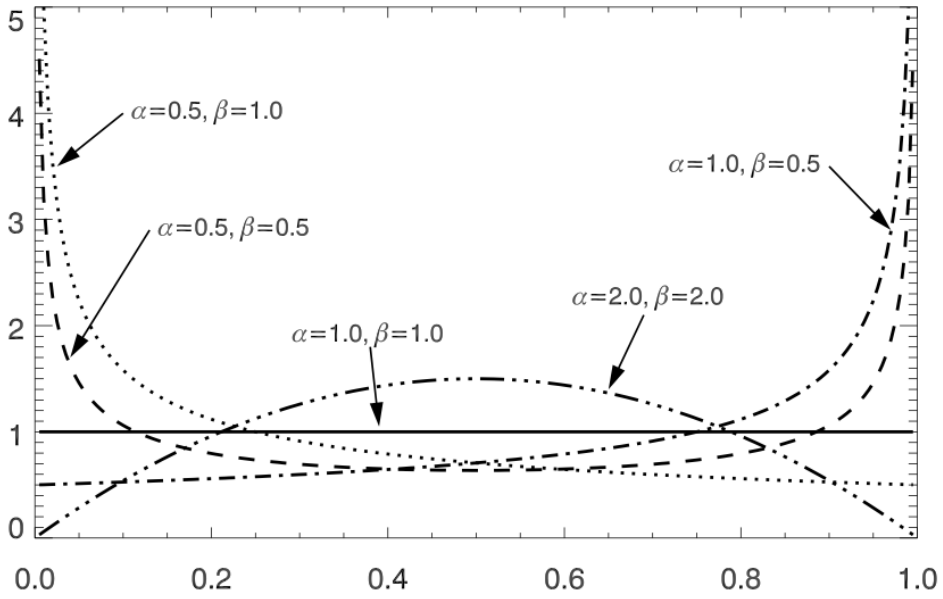


Figure 2: Examples of the probability density of the β-function for different combinations of the parameters α and β.

Bröcker J, Ben Bouallègue Z., **2020**: Stratified rank histograms for ensemble forecast verification under serial dependence. *QJR Meteorol Soc*. 146: 1976-1990.

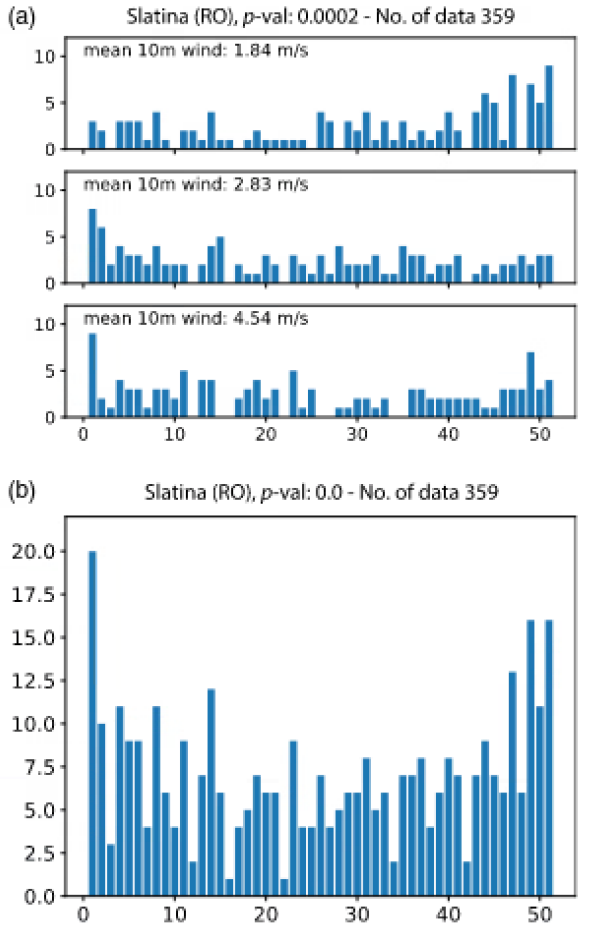


FIGURE 3 (a) Stratified rank histograms, (b) unstratified rank histogram, and (c) corresponding covariance matrix Υ for Salla (Finland). Stratification is based on averaged forecast and observed 2 m temperature. The average of this quantity within the stratum is indicated in each sub-panel of (a). The *p*-value of the reliability test and the sample size (number of forecast–observation pairs) are indicated above (a) and (b). The unstratified histogram shows a warm forecast bias, and the stratified histograms indicate that this is confined to cold conditions [Colour figure can be viewed at wileyonlinelibrary.com]

Allen, S., Ziegel, J., Ginsbourger, D., **2024**: Assessing the calibration of multivariate probabilistic forecasts. *QJR Meteorol Soc*., 1–21.

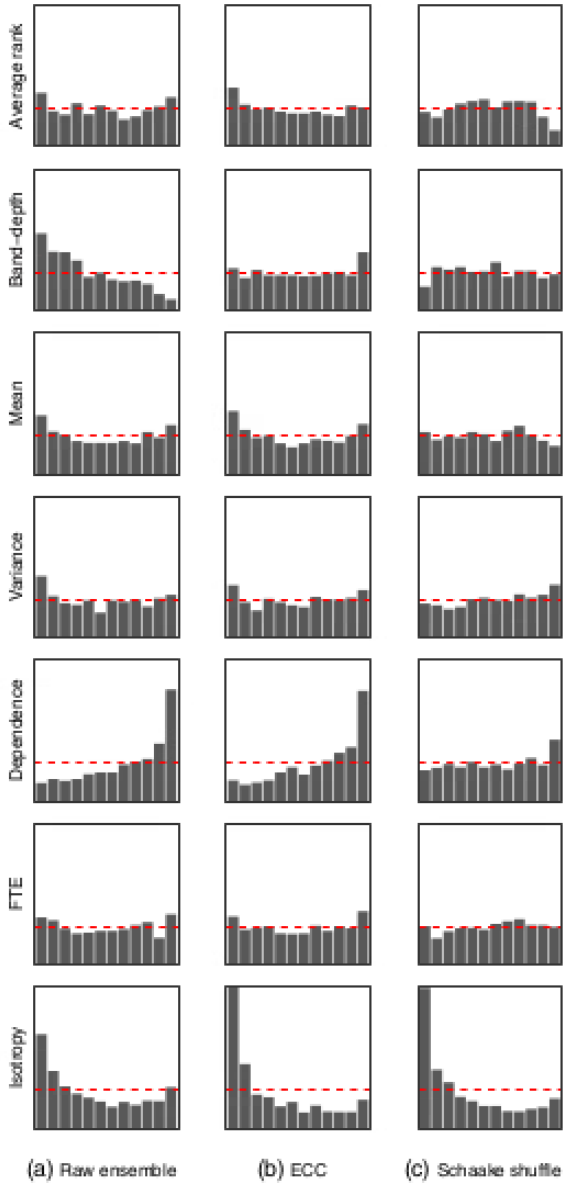


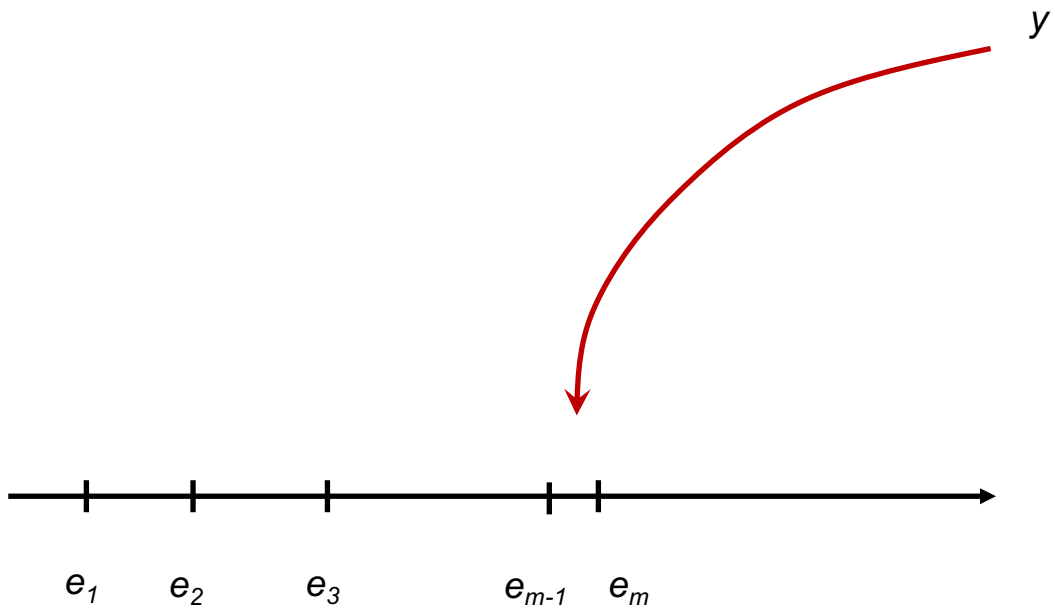
FIGURE 5 Multivariate rank histograms constructed using seven pre-rank functions for (a) the raw ensemble forecasts and (b, c) the post-processed forecasts reordered using (b) ensemble copula coupling (ECC) and (c) the Schaake shuffle. The dashed line indicates a uniform histogram. The *y*-axis limits of all plots are the same. [Colour figure can be viewed at wileyonlinelibrary.com]

3. The observation rank histogram

The ensemble rank histogram

$e_1, e_2, \ldots, e_m$: sorted forecasts over m ensemble members at point (t,s)
y: observation corresponding to forecast e

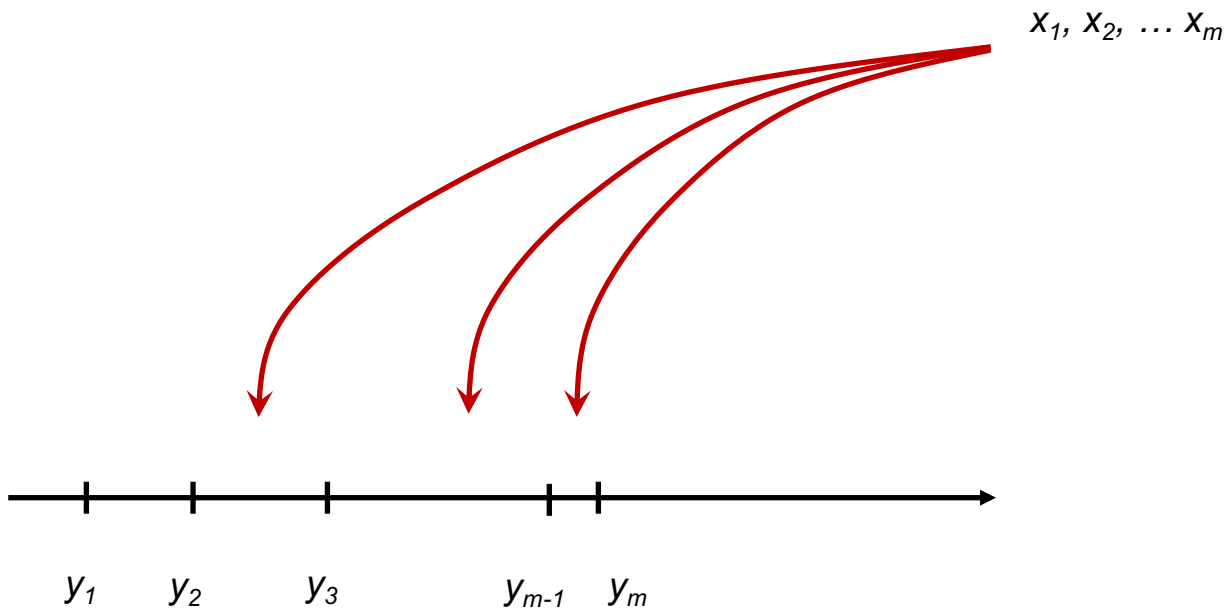
for all t,s



The observation rank histogram

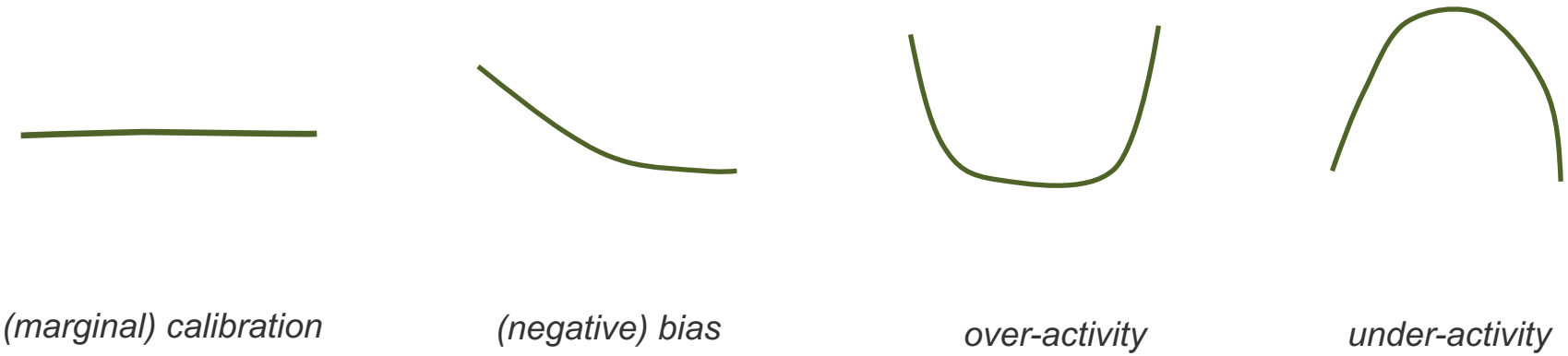
$y_1, y_2, \ldots, y_m$: sorted observation over m dates at station s
$x_1, x_2, \ldots, x_m$: forecasts over m dates at station s

for all s



The observation rank histogram

Interpretation (stand-point = observation) :

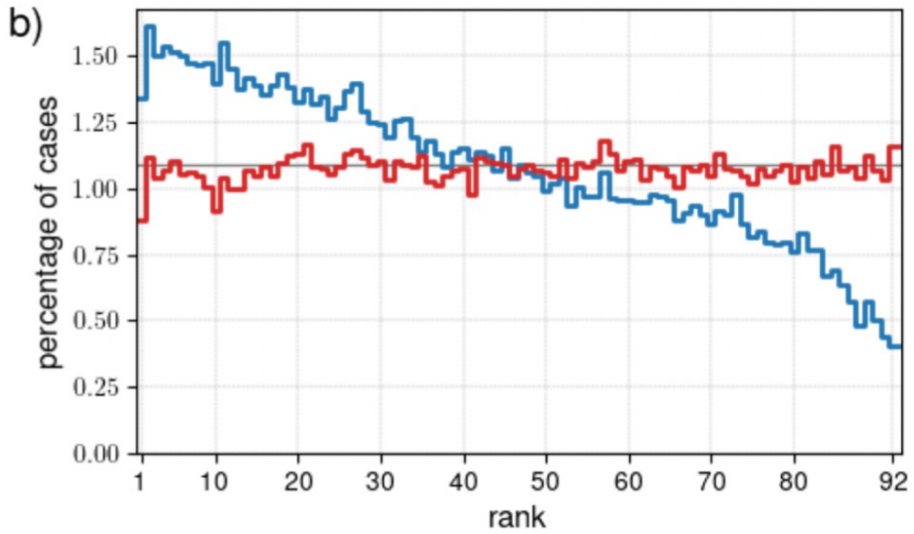


(marginal) calibration *(negative) bias* *over-activity* *under-activity*

The observation rank histogram

Example:

2m temperature, Summer 2022, day 6, Europe
IFS **PanguWeather**



Ben-Bouallegue et al (2024)
10.1175/BAMS-D-23-0162.1

The observation rank histogram

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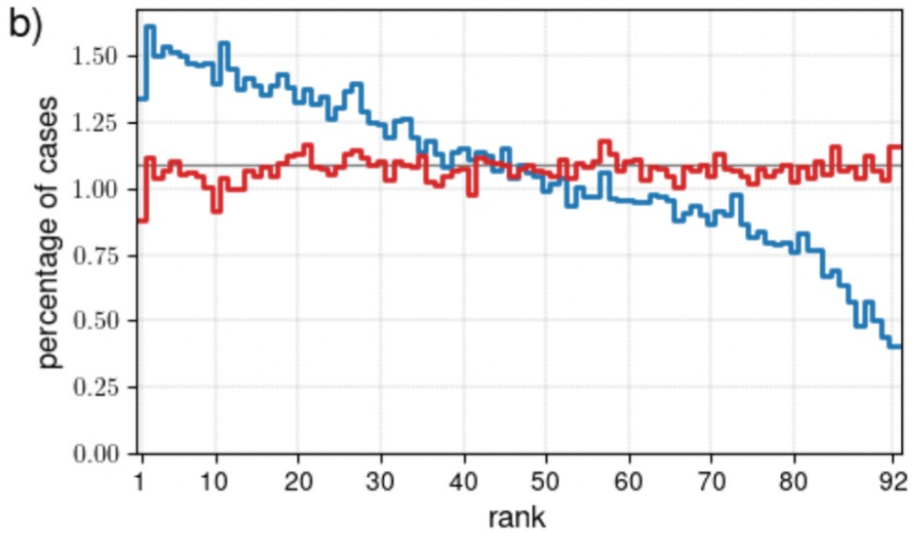

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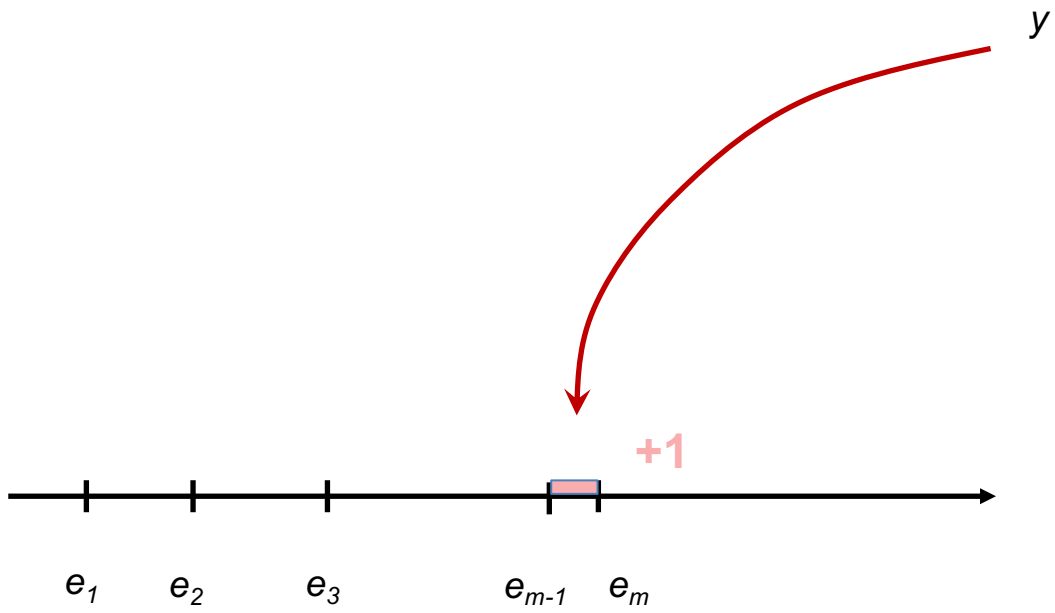
Complementary tool:
Q-Q plot

4. The 2D ensemble rank histogram

The ensemble rank histogram

$e_1, e_2, \ldots, e_m$: sorted forecasts over m ensemble members at point (t,s)
y: observation corresponding to forecast e

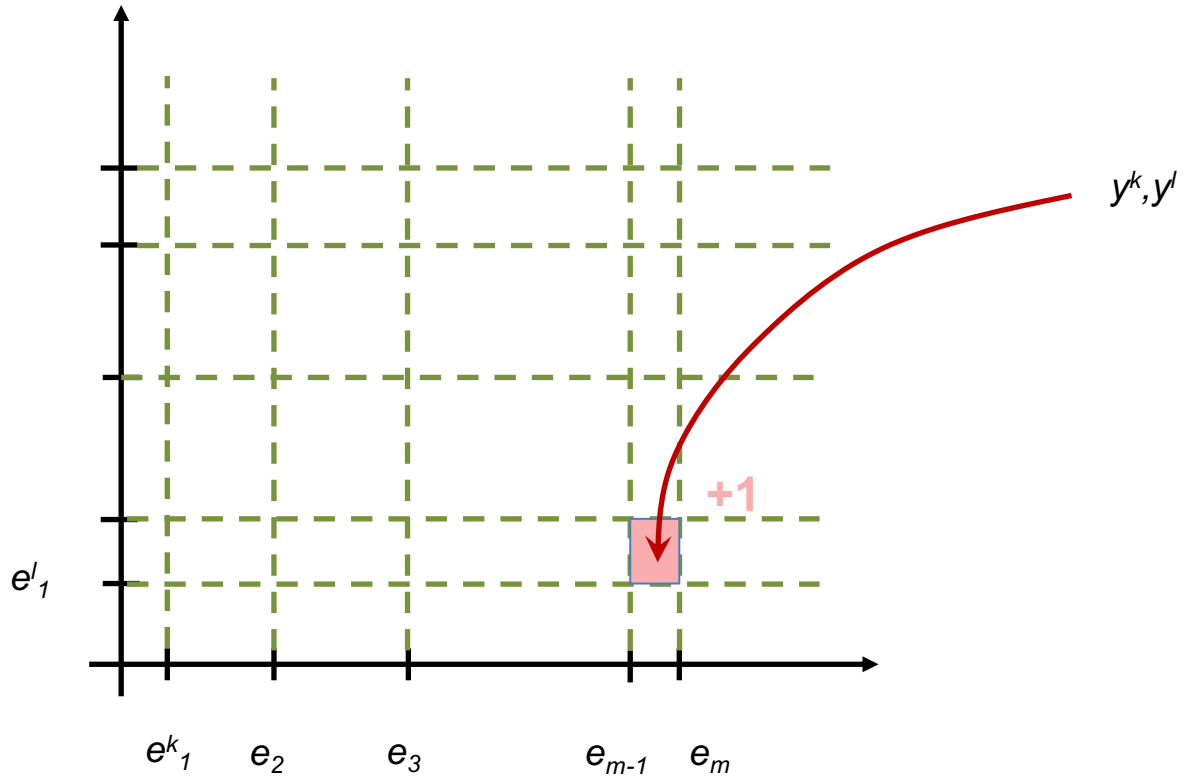
for all t,s



The 2D ensemble rank histogram

$\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_m$: sorted forecasts over m ensemble members at point (t,s)
$\boldsymbol{y}$: observation corresponding to forecast e

for all t,s



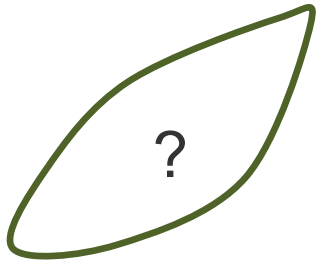
The 2D ensemble rank histogram

Interpretation

in 1D

calibrated

in 2D



it depends! (not known a-priori)

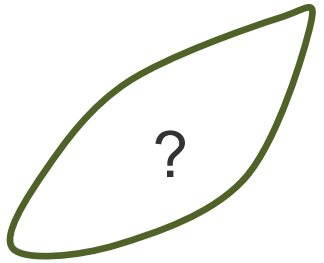
The 2D ensemble rank histogram

Interpretation

in 1D

calibrated

in 2D



it depends! (not known a-priori)

First, we need to get a reference doing the test with the ensemble (**only**) :

$\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_{m-1}$: sorted forecasts over m ensemble members at point (t,s)
$\boldsymbol{e}_m$: pseudo-observation corresponding to member m

for all t,s

The 2D ensemble rank histogram

Interpretation

Sklar's theorem:

***F**: Marginals*

$$G(y_1, ..., y_L) = C(F_1(y_1), F_L(y_L))$$

***G**: Multivariate cumulative distribution function*

***C**: Copula*

The 2D ensemble rank histogram

Examples:

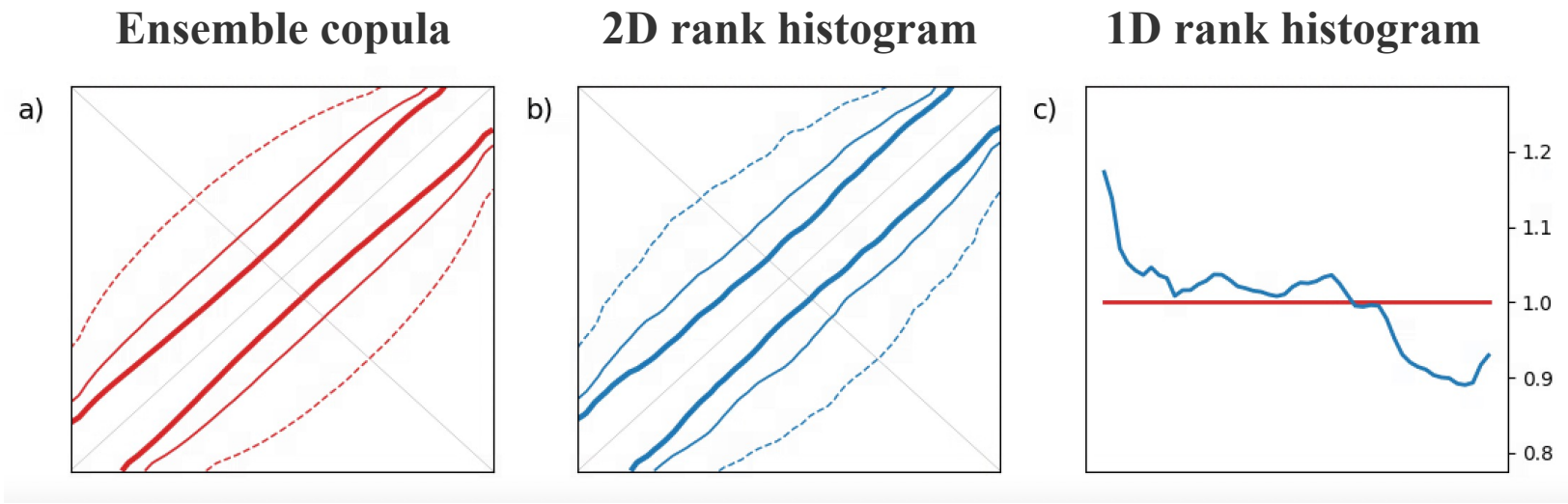
1. Z500 at a zonal distance of 10 deg. ---- **in space**
2. Z500 at day 5 and day 6 ---- **in time**
3. U850 and V850 ---- **intervariable**

IFS ensemble, JJA 2023

The 2D ensemble rank histogram

[stand-point = 2D ensemble]

1. Z500 at a zonal distance of 10 deg. (S. Hem.)



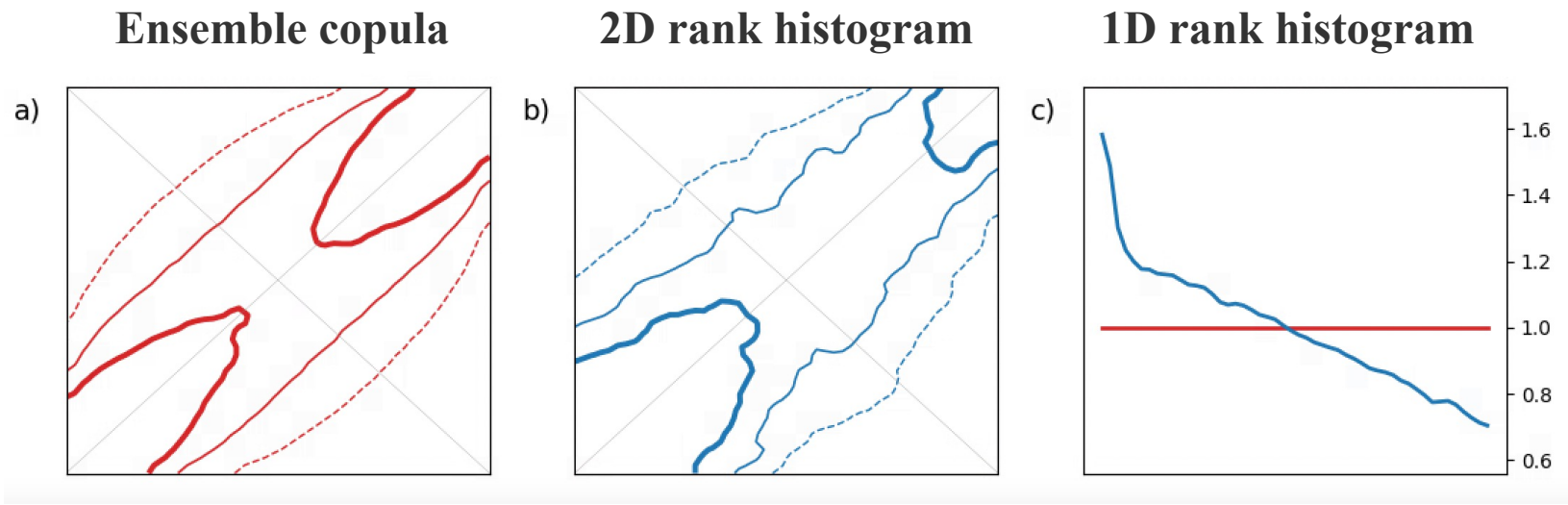
Red: reference

Blue: observation

The 2D ensemble rank histogram

[stand-point = 2D ensemble]

2. Z500 at day 5 and day 6 (N. Hem.)



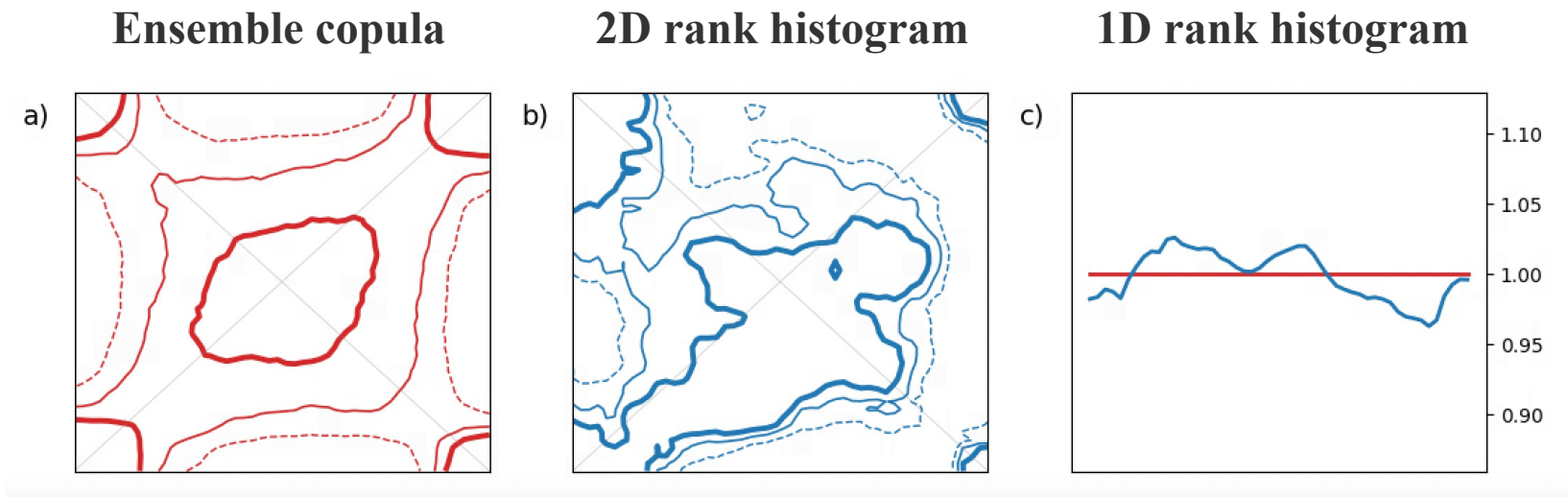
Red: reference

Blue: observation

The 2D ensemble rank histogram

[stand-point = 2D ensemble]

3. U and V at 200hPa (N. Hem.)



Red: reference

Blue: observation

SUMMARY / Outlook

Rank histograms are **versatile tools** for exploring and comparing forecast and observation **statistical structures**,
also in a **multivariate space.**

- **The observation rank histogram** is a complementary tool to the Q-Q plot. It can be used to assess **forecast activity** and forecast ability to capture **observed extremes**.

- **The 2D ensemble rank histogram** is a **generalisation** of the ensemble rank histogram to assess **dependences** in bivariate ensemble forecasts, using the **ensemble copula** as a reference.

- **Outlook:**
  - Statistical **significance test** for 2D rank histograms?
  - **2D observation rank histogram** to compare observation and forecast "climatological" copulas