Met Office

A ‘fair reliability’ diagram for ensemble forecasts

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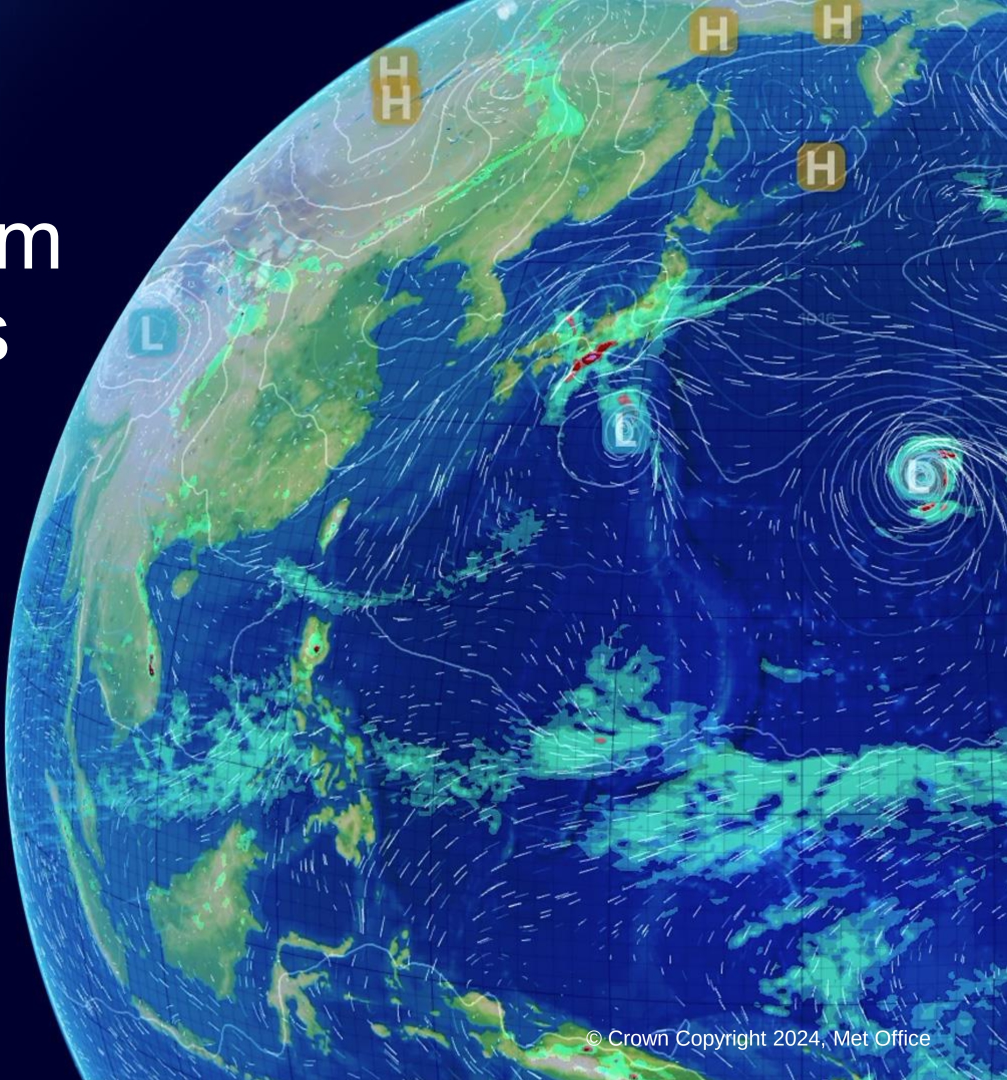


Reliability (calibration) diagrams for *probabilistic* forecasts

For a binary event:

- Group the probability forecasts into bins
- Plot the proportion of times the event occurred when the forecast fell into each bin against a typical probability for that bin.

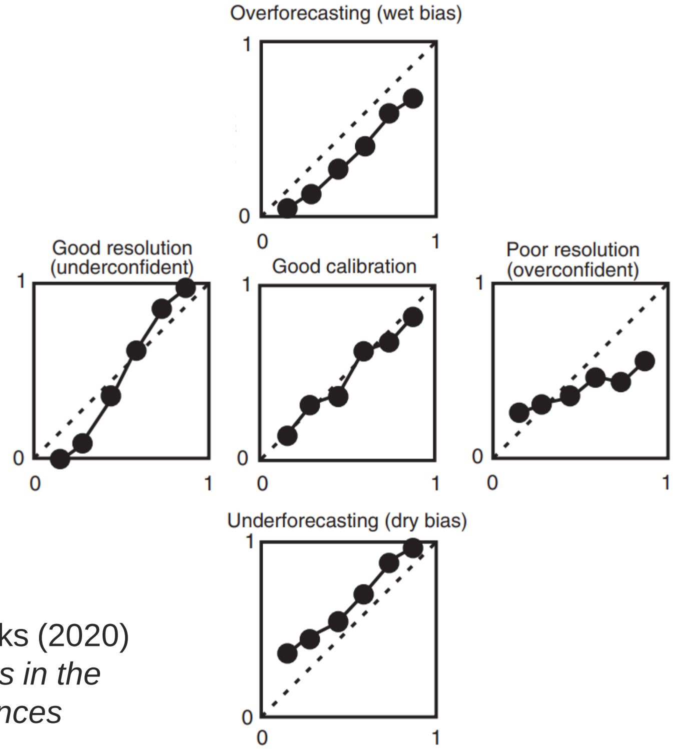


Figure 9.10 of Wilks (2020)
Statistical Methods in the Atmospheric Sciences

Reliability for ensembles

Definition:

An ensemble forecast is *reliable* if the ensemble members and the verifying observation behave as if they have been sampled from the same distribution.

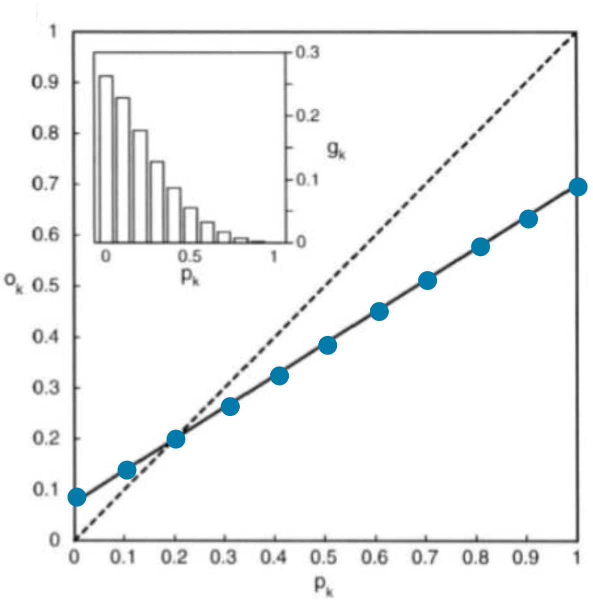
Fair scoring rules measure ensemble performance in a way that favours ensembles that are reliable in this sense

See Ferro, 2014 “Fair scores for ensemble forecasts” *QJRMS.* DOI:10.1002/qj.2270

Reliability diagrams for *ensemble* forecasts

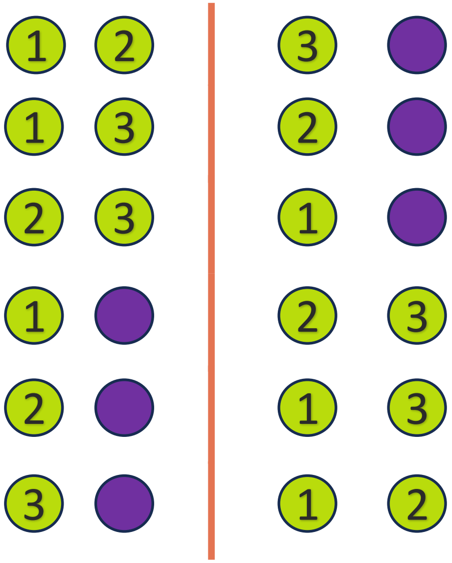
When probability forecasts are derived from the fraction of ensemble members that predict the event, the forecasts appear overconfident when plotted on a reliability diagram *even if the ensemble is perfectly reliable*.

DS Richardson, 2001:
“Measures of skill and value of ensemble prediction systems, their interrelationship and the effect of ensemble size.”
QJRMS 127: 2473-2489



An alternative

- Consider a three-member ensemble forecast *plus its verifying observation* as a single set with four elements.
- If the ensemble is reliable and we have a large number of such sets, of all the sets in which 2 of the 4 elements exceed the threshold, we expect the *observation* to exceed the threshold in half the sets.
- Similarly for 1 and for 3 elements exceeding the threshold.
- Plotting the *actual* fraction of the sets in which the observation exceeds the threshold against these *expected* fractions gives a ‘fair reliability diagram’



Similar ‘trick’ to Bröcker & Ben Bouallègue 2020 “Stratified rank histograms …” *QJRMS*. DOI:10.1002/qj.3778

Sketch of proof

Consider a single ensemble forecast consisting of binary variables $X_1, X_2, \ldots X_m$, and a verifying observation Y. Let $\sum X_i = K$. Condition on a specific value, j, of the sum of the ensemble members and the observation.

Clearly,

$$\mathrm{E}\left(Y + \sum X_i \,\middle|\, Y + \sum X_i = j\right) = j$$

$$\mathrm{E}(Y \mid Y + K = j) + \sum \mathrm{E}(X_i \mid Y + K = j) = j$$

If Y and all the X_i are exchangeable, $\mathrm{E}(Y \mid Y + K = j) = E(X_i \mid Y + K = j)$ for all i.

Therefore

$$(m+1)\,\mathrm{E}(Y \mid Y + K = j) = j$$

$$\mathrm{E}(Y \mid Y + K = j) = \frac{j}{m+1}$$

‘Fair reliability table’ for 3-member ensemble

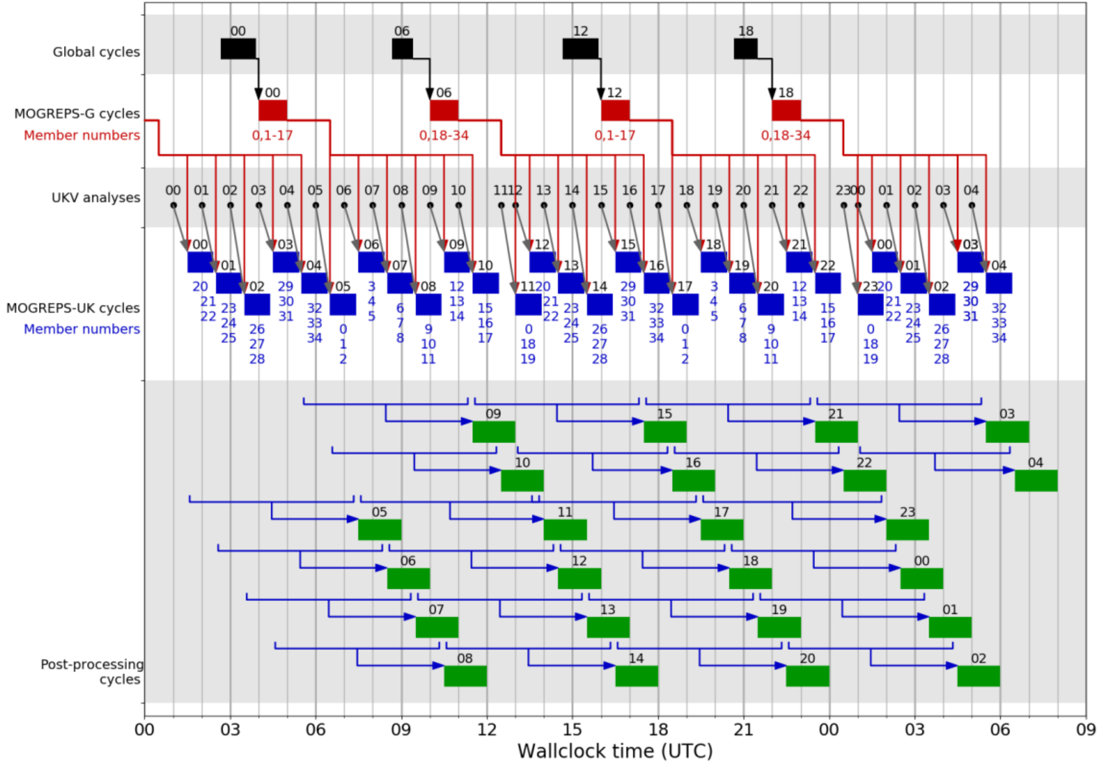
Number of elements in {ens members + obs} with event	1	2	3
Number of events observed	a_0	a_1	a_2
Number of non-events observed	b_1	b_2	b_3
Observed frequency	$a_0/(a_0+b_1)$	$a_1/(a_1+b_2)$	$a_2/(a_2+b_3)$

Example: Hourly-cycling MOGREPS-UK

- *05, 11, 17, 23 UTC cycles*:
  1 control run + 2 perturbed members
- *All other cycles*:
  3 perturbed members

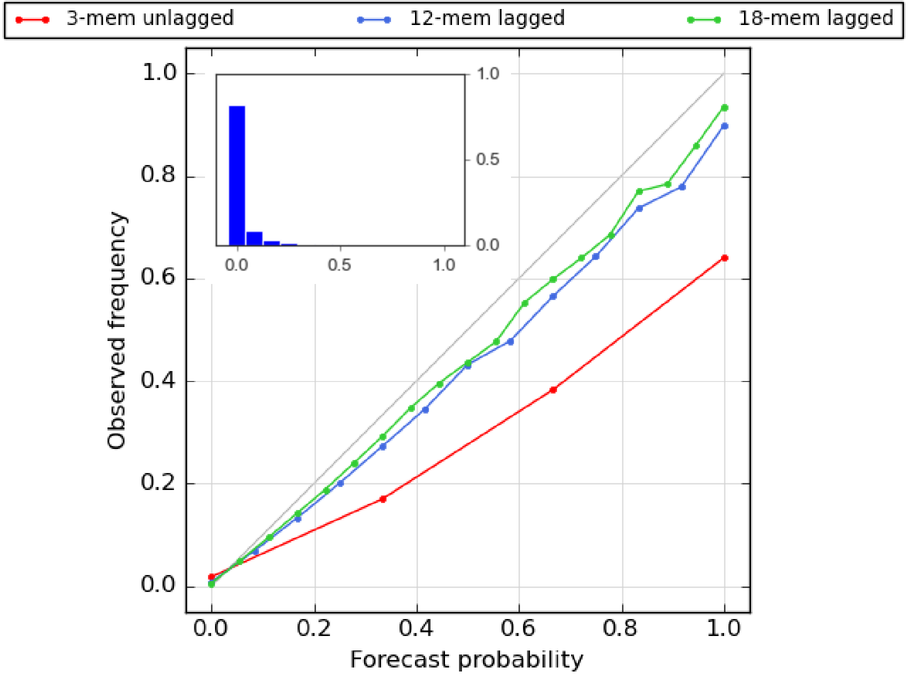
An 18-member ensemble is created by time-lagging over the 6 most recent cycles.

Porson, Carr, Hagelin *et al.*, 2020. “Recent upgrades to the Met Office convective-scale ensemble: an hourly time-lagged 5-day ensemble.” *QJRMS*. DOI:10.1002/qj.3844

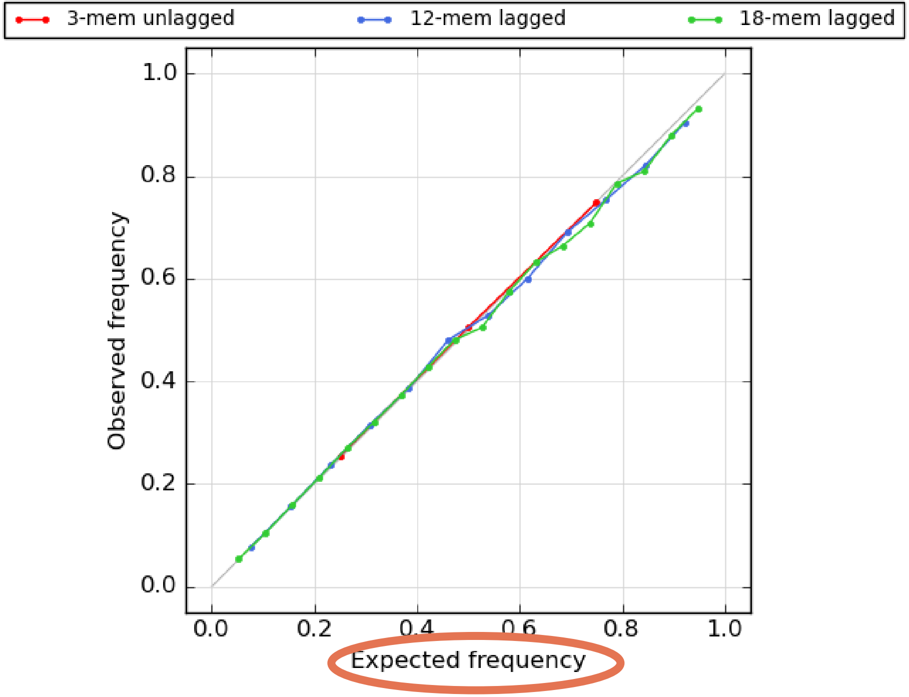


1-hr precip accumulation ⩾1 mm, T+24

Conventional reliability diagram

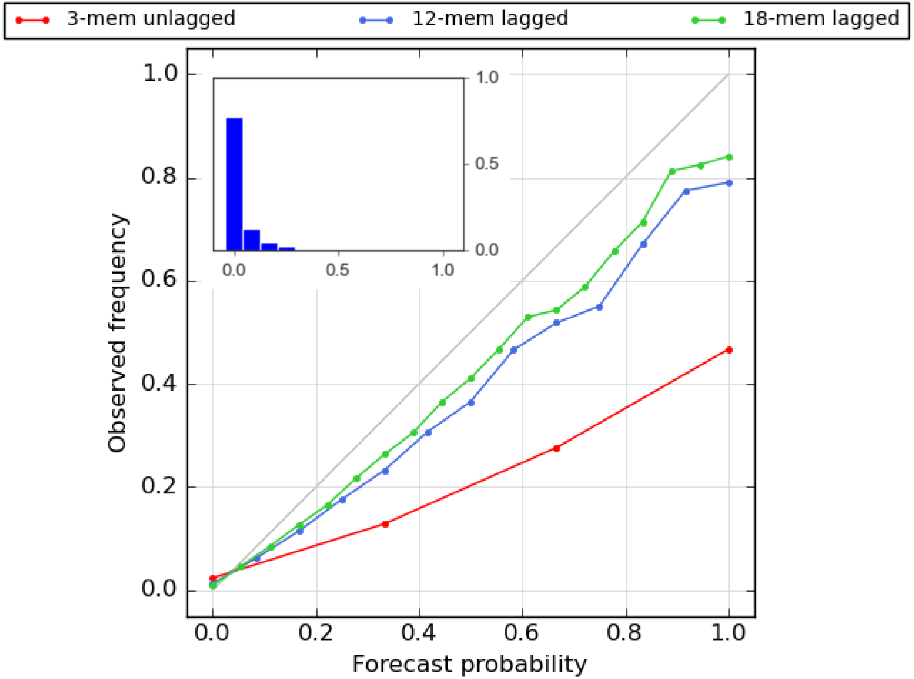


‘Fair reliability’ diagram

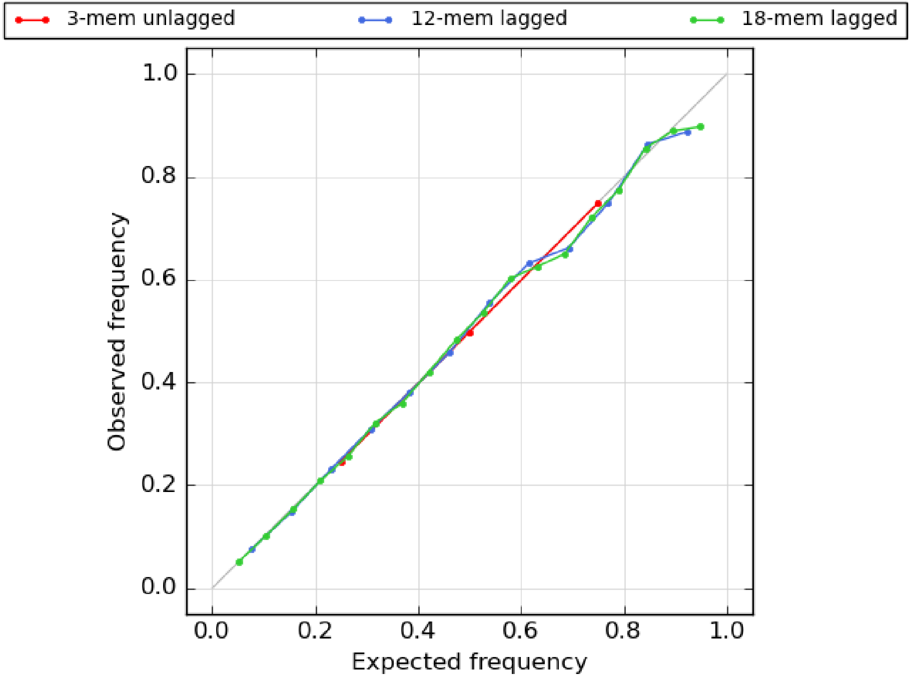


1-hr precip accumulation ⩾1 mm, **T+72**

Conventional reliability diagram



Fair reliability diagram



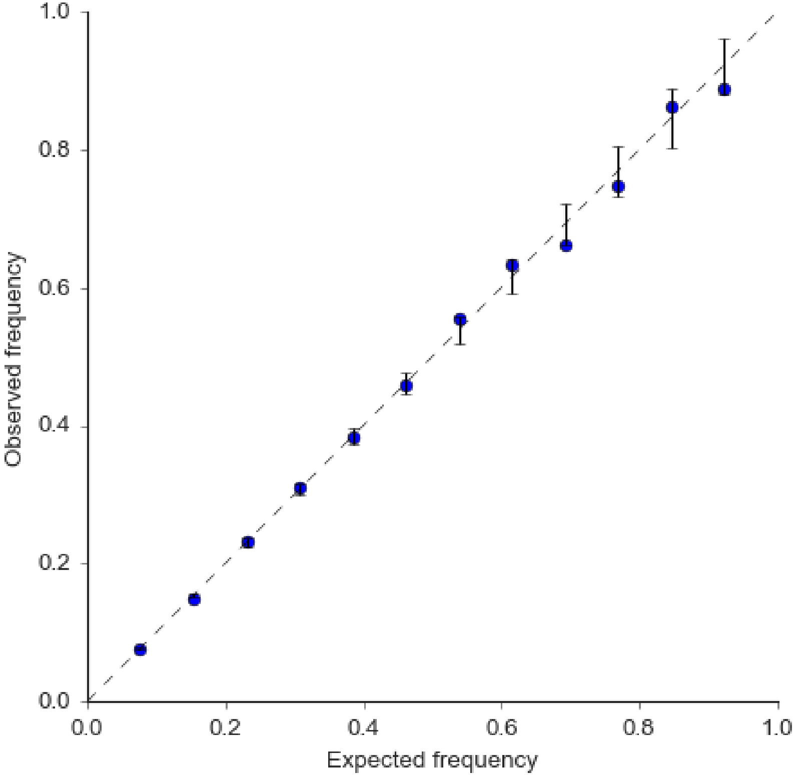
Consistency bars

‘Fair reliability diagram

Similar to Bröcker & Smith, 2007.
*“*Increasing the reliability of reliability diagrams.*”* *Weather and Forecasting* 22: 651-661. DOI:10.1175/WAF993.1

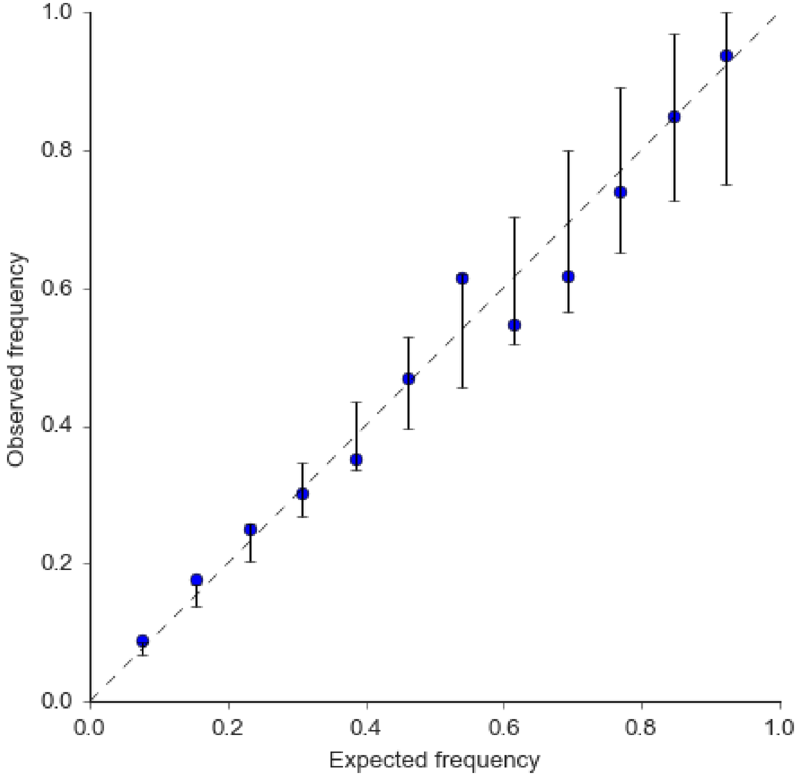
90% interval around the diagonal, computed using binomial percentiles.

Here showing T+72, 12-member lagged ensemble for all of 2020-2022 using 8 cycles per day



Consistency bars

Fair reliability diagram



90% interval around the diagonal, computed using binomial percentiles.

Here showing T+72, 12-member lagged ensemble for *DJF 2023-24 using 4 cycles per day* – i.e. **much smaller sample size**

Summary measures of reliability

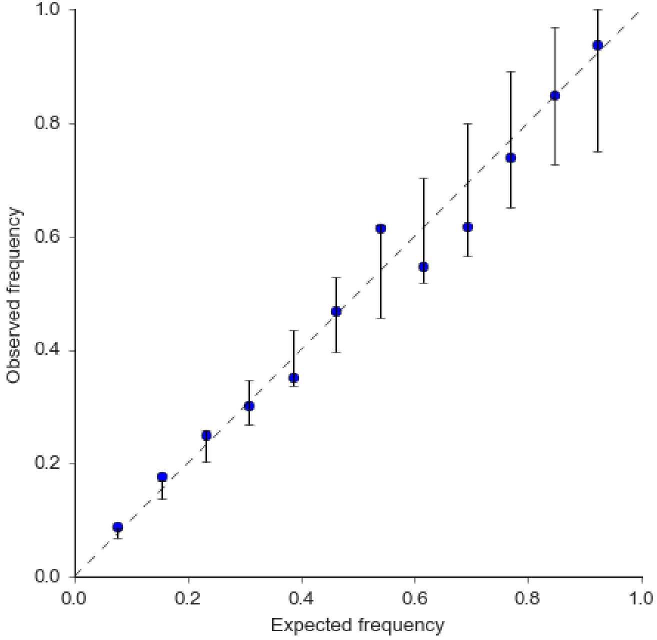
- The reliability component of the Brier score can be viewed a weighted sum of squared distances from the diagonal of the conventional reliability diagram.
- This also can be misleading for small ensembles.
- Calculating an analogous quantity for the *fair* reliability diagram could give a summary measure of ensemble reliability (‘ensemble miscalibration’).

- Might this lead to a decomposition of the *fair* Brier score…?
- It appears not, unfortunately 😟
- So maybe ‘fair reliability diagram’ isn’t a good name... but what’s a better one?

Summary

- Conventional reliability (calibration) diagrams are misleading for small ensembles
- Including the verifying observation in the conditioning overcomes this, giving a ‘fair reliability’ diagram
- Consistency bars aid interpretation

- Work in progress – feedback appreciated!



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